(Introduction to Fractions)

Ground Rules for Problem Set Completion

- 1. Present your work in a neat and organized manner. Use <u>complete sentences</u> whenever you are asked to make a statement.
- 2. SHOW YOUR WORK: Credit is awarded for all reasonable attempts based on the work shown.
- 3. Make sure you answer ALL parts of problems.
- 4. Complete and submit ALL Problem Sets for the unit prior to taking the Unit Test.
- I. WHAT ARE FRACTIONS?

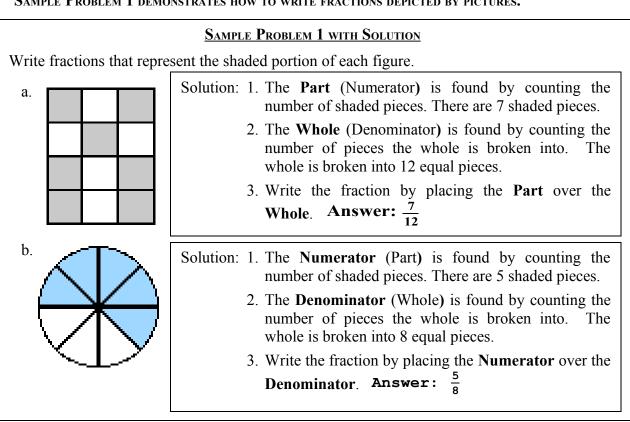
Fractions are numbers used to represent part of something. For example, a yard is made up of 3 feet. Thus, a foot is $\frac{1}{3}$ (one-third) of a yard.

One way to think of a fraction is as $\frac{Part}{Whole}$, where **Part** is the number of equal portions we are concerned with and **Whole** is the number of equal portions the original whole unit was broken into. In the example in the first paragraph, the **Part** is 1 (since we are concerned with just 1 foot) and the **Whole** is 3 (since the original whole yard can be broken into 3 equal feet).

In mathematics, the **Part** of the fraction is called its **Numerator** and the **Whole** of the fraction is called its **Denominator**.

The equation below summarizes the discussion above.

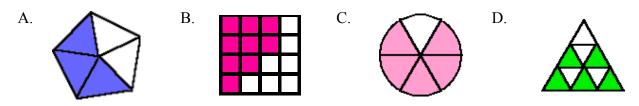
Numerator	_ Part	Number of equal parts we are concerned with
Denominator	Whole	= Number of equal parts in the original whole



SAMPLE PROBLEM 1 DEMONSTRATES HOW TO WRITE FRACTIONS DEPICTED BY PICTURES.

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For problems A through D, write a fraction that represents the shaded portion of each figure. Refer to Sample Problem 1 on the previous page, as needed. For more practice, see page 11 of Contemporary's <u>Number Power 2</u> work-text.



SAMPLE PROBLEM 2 DEMONSTRATES HOW TO WRITE FRACTIONS FROM WORD DESCRIPTIONS.

Samp	PLE PROBLEM 2 WITH SOLUTION
Write fractions for each of the parts	described below.
a. A liter contains 1000 milliliters. What fraction of a liter is 323 milliliters?	 Solution: 1. The Whole (Denominator) is the liter, which consists of 1000 equal milliliters. 2. The Part (Numerator) is the 323 milliliters we are concerned with. 3. Write the fraction by placing the Part over the Whole. Answer: 323/1000
 b. Tim and Sally borrowed \$5,800 to buy a used car. So far, they've paid off \$1,673 of the principal. What fraction of the loan have they paid off? 	 Solution: 1. The Numerator (Part) is the \$1,673 they have paid off. 2. The Denominator (Whole) is the original \$5,800 they borrowed. 3. Write the fraction by placing the Numerator over the Denominator. Answer: 1,673/5,800

For problems E through H, write fractions for each of the *parts* described below. Refer to Sample Problem 2 above, as needed. For more practice, see page 12 of Contemporary's <u>Number Power 2</u> work-text.

- E. What fraction of a 30-day month is 11 days?
- F. There are 5,280 feet in a mile. What fraction of a mile is 1,313 feet?
- G. There are 27 students in Amy's class. If there are 16 girls in Amy's class, what fraction of the class is girls?
- H. Jerry's biweekly take-home pay is \$961. Of this, he deposits \$75 in savings. What fraction of his pay does Jerry deposit in savings.

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II. FORMS OF FRACTIONS

There are three forms of fractions: **proper fractions**, **improper fractions**, and **mixed numbers**.

Proper fractions are those that are most familiar to most of us. A proper fraction <u>always</u> has a smaller numerator (part) than denominator (whole). Examples of **proper fractions** include such numbers as $\frac{1}{2}$, $\frac{4}{5}$, $\frac{289}{312}$.

The prefix "**im**" means "**not**." Thus, an **improper fraction** is <u>not proper</u>. In other words, the numerator (part) of an **improper fraction** is <u>always the same as or more than the denominator</u> (whole). Examples of **improper fractions** include such numbers as $\frac{5}{2}$, $\frac{11}{5}$, and $\frac{289}{31}$.

A mixed number is formed by combining (mixing) a whole number with a proper fraction. Examples of mixed numbers include such numbers as $3\frac{3}{4}$, $1\frac{1}{5}$, and $28\frac{9}{31}$.

SAMPLE PROBLEM 3 DEMONSTRATES HOW TO IDENTIFY THE THREE FORMS OF FRACTIONS.

	SAMPLE PROBLEM 3 WITH SOLUTION							
	Tell whether each of the following is a proper fraction, improper fraction, or a mixed number.							
	<u>18</u> 11	Solution: $\frac{18}{11}$ is an improper fraction since the numerator is as large or larger than the denominator.						
b.	5 12	Solution: $\frac{5}{12}$ is a proper fraction since the numerator is smaller than the denominator.						
c.	$6\frac{2}{3}$	Solution: $6\frac{2}{3}$ is a mixed number since it consists of both a whole number (6) and a fraction $(\frac{2}{3})$.						

For problems A through H, tell whether each number is a **proper fraction**, **improper fraction**, or a **mixed number**. Refer to Sample Problem 3 above, as needed. For more practice, see page 13 of Contemporary's <u>Number Power 2</u> work-text.

A.	<u>5</u> 8	B.	$2\frac{7}{10}$	C.	<u>19</u> 20	D.	<u>12</u> 5
E.	$11\frac{4}{7}$	F.	220 100	G.	<u>45</u> 50	H.	$\frac{11}{11}$

As seen in the discussion above, both **improper fractions** and **mixed numbers** represent numbers equal to or greater than one. We often have to convert between these two forms, depending on how the number used.

To convert an **improper fraction** to a **mixed number** divide the numerator (top) by the denominator (bottom). The number of times the denominator goes into the numerator of the improper fraction becomes the whole number portion of the mixed number. The remainder becomes the numerator of the fraction portion of the mixed number. And the denominator of the improper fraction is also the denominator of the fraction portion of the mixed number.

SAMPLE PROBLEM 4 ON THE NEXT PAGE DEMONSTRATES HOW TO CHANGE IMPROPER FRACTIONS TO MIXED NUMBERS.

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	SAMPLE PROBLEM 4 WITH SOLUTION	
-	ch improper fraction to a whole number or a mixed numbe	r
a. <u>23</u> 8	 Solution: 1. Divide the numerator (top) by the denominator (bottom). 2. The number of times the denominator goes into the numerator is the whole number portion of the mixed number. 3. The remainder is the numerator of the fraction portion of the mixed number. 	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
b. <u>15</u> 5	 4. The denominator of the improper fraction is also the denominator of the fraction portion of the mixed number. Solution: 1. Divide the numerator (top) by the denominator (bottom). 2. Since there is no remainder, the answer is a whole number. 	$ \begin{array}{c} 3 \\ 5)15 \\ \underline{15} \\ 0 \\ \text{Ans. 3} \end{array} $

For problems I through N, change each **improper fraction** to a **whole number** or a **mixed number**. Refer to Sample Problem 4 above, as needed. For more practice, see pages 19 & 20 of Contemporary's <u>Number Power 2</u> work-text.

I.	<u>35</u> 8	J.	50 10	K.	69 20
L.	<u>12</u> 5	M.	<u>49</u> 7	N.	356 55

TO CONVERT A MIXED NUMBER TO AN IMPROPER FRACTION:

- 1. Multiply the denominator (bottom) of the fraction portion by the whole number portion of the mixed number. (*This tells us how many pieces the whole number portion can be broken into, based on the denominator.*)
- 2. Add this result to the original numerator of the fraction portion of the mixed number. (*This gives the total number of pieces we would have if we broke each whole into the number of parts indicated by the denominator.*)
- 3. Write the improper fraction using the result from Step 2 as the numerator of the improper fraction and the original denominator as the denominator of the improper fraction.

For problems O through T, change each **mixed number** to an **improper fraction**. Refer to Examples 1 and 2 on page 21 of Contemporary's <u>Number Power 2</u> work-text, as needed. For more practice, see page 21 of Contemporary's <u>Number Power 2</u> work-text.

0.	3 <u>5</u> 8	Р.	$5\frac{7}{10}$	Q.	$6\frac{9}{20}$
R.	$1\frac{2}{5}$	S.	$4\frac{3}{7}$	Τ.	$2\frac{6}{55}$
			$\mathbf{D}_{\mathbf{\sigma}}$ (of 10		

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III. WRITING EQUIVALENT FRACTIONS

As the name implies, **equivalent fractions** are fractions of equal value. For example, the fraction $\frac{1}{2}$ has the same value as the fraction $\frac{2}{4}$. We see this is true if we think about money; since, a half-dollar is worth the same as 2 quarters. Both are worth 50 cents.

When working with equivalent fractions there are two tasks that we need to be able to do: (1) Reduce fractions to **lowest terms** and (2) raise fractions to higher terms.

Normally, answers to fraction problems are given in lowest terms.

	SAMPLE PROBLEM 5 WITH SOLUTION									
$\frac{60}{75}$	Method 1: Divide the numerator and denominator by any factor they have in common. Repeat until there are no common factors remaining.									
to lowest terms.	$\frac{60 \div 5}{75 \div 5} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5} \text{ or, if you prefer, } \frac{60}{75} \div \frac{5}{5} = \frac{12}{15} \div \frac{3}{3} = \frac{4}{5}$									
	Method 2: Divide the numerator & denominator by the Greatest Common Factor.									
	$\frac{60 \div 15}{75 \div 15} = \frac{4}{5}$ or, if you prefer, $\frac{60}{75} \div \frac{15}{15} = \frac{4}{5}$									

For problems A through H, reduce each fraction to **lowest terms**. Refer to Sample Problem 5, above, and Examples 1 through 4 on pages 15 & 16 of Contemporary's <u>Number Power 2</u> work-text, as needed. For more practice, see pages 16 and 17 of Contemporary's <u>Number Power 2</u> work-text.

A.	$\frac{50}{80}$	B.	72 120	C.	<u>33</u> 66	D.	<u>12</u> 60
E.	<u>34</u> 51	F.	<u>180</u> 200	G.	<u>35</u> 50	H.	27 81

In addition to reducing fractions to **lowest terms** we need to be able to raise fractions to **higher terms**. Normally, raising fractions to **higher terms** is used when we need to have common denominators for comparing, adding, and subtracting fractions.

SAMPLE PROBLEM 6 WITH SOLUTION

Create equivalent fractions by raising the fraction to the specified higher term $\frac{3}{8} = \frac{?}{48}$.

Solution: Divide the larger denominator by the smaller denominator, then multiply both the numerator and denominator of the original fraction by this factor. *(This works because we are simply multiplying the original fraction by 1.)*

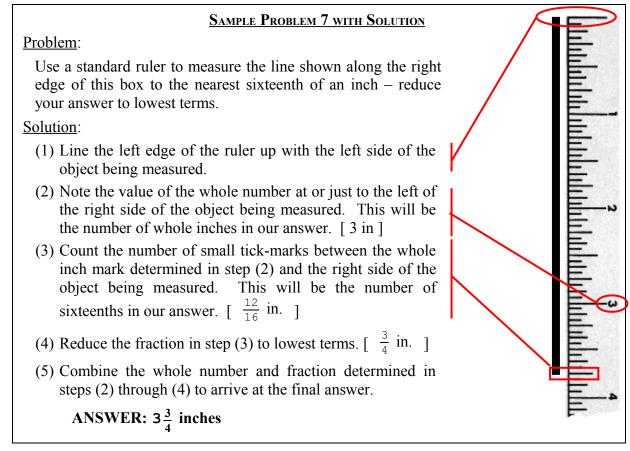
 $\frac{3*6}{8*6} = \frac{18}{48}$ or, if you prefer, $\frac{3}{8} * \frac{6}{6} = \frac{18}{48}$

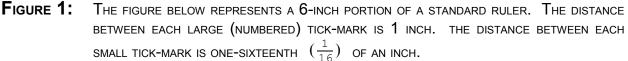
For problems I through N, create **equivalent fractions** by raising each fraction to the specified **higher term**. Refer to Sample Problem 6, above, and Examples 1 and 2 on page 18 of Contemporary's <u>Number Power 2</u> work-text, as needed. For more practice, see page 18 of Contemporary's <u>Number Power 2</u> work-text.

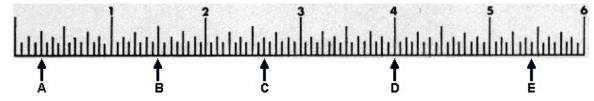
I.	$\frac{1}{2} * - = \frac{?}{10}$	J.	$\frac{3}{4} * - = \frac{?}{36}$	K.	$\frac{7}{10} * - = \frac{?}{60}$
L.	$\frac{5}{12} * - = \frac{2}{48}$	M.	$\frac{9}{13} * - = \frac{?}{78}$	N.	$\frac{11}{21} * - = \frac{?}{63}$

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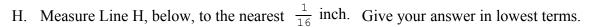
IV. READING A STANDARD RULER

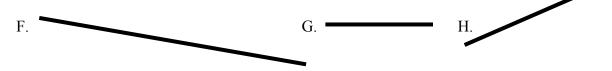






- A-E. <u>Using Figure 1 above</u>, state how far each of the indicated points is from the left edge of the ruler. <u>Give your answers to the nearest sixteenth of an inch</u>. For more practice, see page 134 of Contemporary's <u>Number Power 2</u> work-text.
 - F. Measure Line F, below, to the nearest $\frac{1}{4}$ inch. Give your answer in lowest terms.
- G. Measure Line G, below, to the nearest $\frac{1}{8}$ inch. Give your answer in lowest terms.





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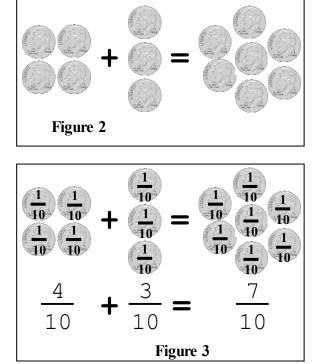
V. Adding & subtracting fractions with like denominators

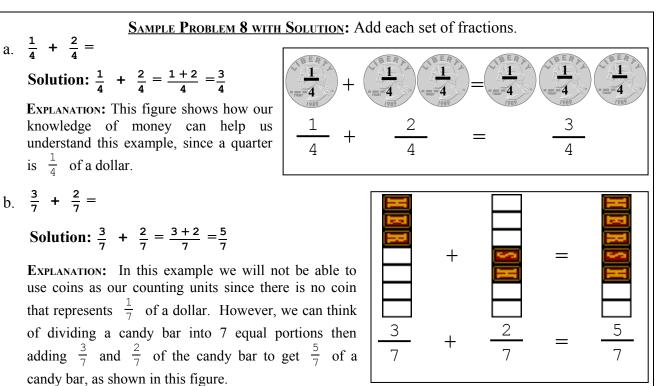
Most of us learned to add whole numbers by "**counting up.**" For example, if we refer to the Figure 2, we see that we are able to determine that "4 dimes plus 3 dimes" adds up to 7 dimes by counting the dimes in each pile.

Adding fractions can also be done by "counting up," so long as we are counting the same size pieces. That is to say that the fractions we are adding must have the same denominators.

We can use the above example of "4 dimes plus 3 dimes equals 7 dimes" to illustrate how this works with fractions. We know that there are ten dimes in a dollar. Thus, each dime in Figure 2 represents one-tenth $\left(\frac{1}{10}\right)$ of a dollar.

Figure 3 shows how, if we think of each dime as $\frac{1}{10}$ a dollar, we can count up the tenths and see that " $\frac{4}{10}$ plus $\frac{3}{10}$ equals $\frac{7}{10}$."





For problems A - F, add each set of fractions and reduce. Refer to Sample 8, as needed. For more practice, see pages 22 - 24 of Contemporary's <u>Number Power 2</u> work-text.

A. $\frac{1}{4} + \frac{2}{4} =$ B. $\frac{2}{5} + \frac{1}{5} =$ C. $\frac{1}{8} + \frac{5}{8} =$ D. $\frac{3}{10} + \frac{5}{10} =$ E. $\frac{5}{12} + \frac{3}{12} + \frac{1}{12} =$ F. $\frac{15}{100} + \frac{13}{100} + \frac{2}{100} =$ Pg. 7 of 10Created by R. E. Buzby, Sidney, ME -- 1998-2010

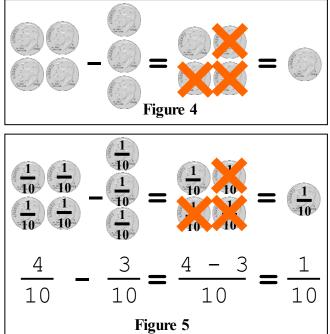
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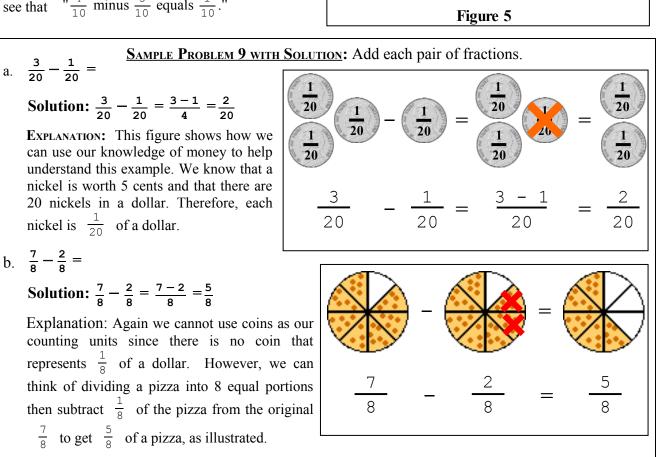
Just as we can "**count up**" to add, we can "**count down**" to subtract. For example, if we refer to the Figure 4, we can see that "4 dimes minus 3 dimes equals 1 dime" by counting the dime(s) left in the original pile after removing the dimes being subtracted. (This is why subtraction is often thought of as "**Take away**."

Subtracting fractions can be done by "counting down" in exactly the same way that adding fractions can be done by "counting up," so long as the fractions we are adding or subtracting have the same denominators.

We can use the above example of "4 dimes minus 3 dimes equals 1 dime" to illustrate how this works with fractions.

Figure 5 shows how, if we think of each dime as $\frac{1}{10}$ of a dollar, we can count down the tenths & see that $\frac{4}{10}$ minus $\frac{3}{10}$ equals $\frac{1}{10}$."





For problems G - L, subtract each pair of fractions and reduce. Refer to Sample 8, as needed. For more practice, see page 31 of the <u>Number Power 2</u> work-text.

G. $\frac{4}{5} - \frac{2}{5} =$ H. $\frac{3}{4} - \frac{1}{4} =$ I. $\frac{7}{8} - \frac{5}{8} =$ J. $\frac{9}{10} - \frac{3}{10} =$ K. $\frac{11}{12} - \frac{7}{12} =$ L. $\frac{73}{100} - \frac{48}{100} =$ Pg. 8 of 10Created by R. E. Buzby, Sidney, ME -- 1998-2010

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VI. SOLVING ADDITION & SUBTRACTION PROBLEMS INVOLVING FRACTIONS WITH LIKE DENOMINATORS

Probably the most important mathematics skill is that of problem solving. If you cannot solve problems, then all your other math skills cannot be used to their fullest. Consequently, throughout this course you will be required to apply the skills you learned to solving simple problems.

To help you focus your attention and develop your problem-solving skills, you will required to use the three steps below whenever you solve a problem.

- 1. State what it is you are to find. Give your answer as a complete sentence.
- 2. Solve the problem, showing your work.
- 3. State the answer in a complete sentence.

SAMPLE PROBLEM 10 DEMONSTRATES HOW TO USE THESE STEPS TO SOLVE A PROBLEM.

SAMPLE PROBLEM 10 WITH SOLUTION

The Problem:

Joel is making an outfit that calls for $2\frac{7}{8}$ yards of material. If he already has

 $1\frac{3}{8}$ yards on hand, how much more material does he need to buy?

The Solution:

- a. We are to find how much additional material Joel has to buy.
- b. To find the additional material needed, subtract the material on hand $(1\frac{3}{8} \text{ yd})$ from the amount required to make the outfit $(2\frac{7}{8} \text{ yd})$. One estimate is $1\frac{1}{2}$ yards $(3-1\frac{1}{2})$ ate is $\pm \frac{1}{2}$ yards \sum_{2}^{-2} The calculation at the right $2\frac{7}{8}$ gives the exact amount needed. $-\frac{1\frac{3}{8}}{1\frac{4}{8}} = 1\frac{1}{2}$ yards

COMPLETELY SOLVE PROBLEMS A – D USING STEPS (1), (2), AND (3) BELOW. REFER TO SAMPLE PROBLEM 10 AND THE INTRODUCTION TO PROBLEM SOLVING HANDOUT, AS NEEDED.

- 1. State what it is you are to find. Give your answer as a complete sentence.
- 2. Solve the problem, showing your work. Reduce all fractions & mixed numbers to lowest terms.
- 3. State the answer in a complete sentence.
- A. After the class's pizza party Jerry wanted to know how much pizza was left over. He found that there was $\frac{3}{8}$ of a pizza left at Team 1's table, $\frac{2}{8}$ of a pizza left at Team 2's table, and $\frac{1}{8}$ of a pizza left at Team 3's table. In all, how much pizza was left?
- B. Linda is making a dress with a matching vest. She needs $2\frac{1}{8}$ yards of material for the dress and $\frac{5}{8}$ of a yard for the vest. How much material does she need in all?
- C. The recipe calls for $2\frac{2}{3}$ cups of milk. Tim has already stirred in $1\frac{1}{3}$ cups. How much more milk should he add to the mix?
- D. Ernie starts with an aluminum bar that is $7\frac{7}{8}$ inches long. What is the length of bar left after Ernie cuts off a piece $2\frac{5}{8}$ inches long?

(Introduction to Fractions)

Answer Key

Section I: What are fractions?										
A. $\frac{3}{5}$	B.	9 16	C.	<u>5</u> 6	D.	<u>5</u> 9				
E. $\frac{11}{30}$	F.	1,313 5,280	G.	<u>16</u> 27	H.	75 961				
Section II: Forms of fractions										
A. Proper fraction	В.	Mixed number	C. 1	Proper fraction	D. 1	Improper fraction				
E. Mixed number	F.	Improper fraction	G. 1	Proper Fraction	H. 1	Improper fraction				
I. $4\frac{3}{8}$	J.	5	K.	$3\frac{9}{20}$	L.	$2\frac{2}{5}$				
M. 7	N.	6 <u>26</u> 55	0.	<u>29</u> 8	P.	57 10				
Q. $\frac{129}{20}$	R.	<u>7</u> 5	S.	<u>31</u> 7	T.	116 55				
SECTION III: WRITING EQUIV	ALENI	FRACTIONS								
A. $\frac{5}{8}$	B.	<u>3</u> 5	C.	$\frac{1}{2}$	D.	$\frac{1}{5}$				
E. $\frac{2}{3}$	F.	<u>9</u> 10	G.	$\frac{7}{10}$	H.	$\frac{1}{3}$				
I. $\frac{5}{10}$	J.	27 36	K.	<u>42</u> 60	L.	20 48				
M. $\frac{54}{78}$	N.	<u>33</u> 63								
SECTION IV: READING A STAN	NDARD	RULER								
A. $\frac{1}{4}$ "	B.	$1\frac{1}{2}$ "	C.	2 ⁵ / ₈ "	D.	4 "				
E. $5\frac{7}{16}$ "	F.	2 3 "	G.	1 1 "	H.	$1\frac{1}{2}$ "				
Section V: Adding & sub	RACT	ING FRACTIONS WITH	LIKE D	DENOMINATORS						
A. $\frac{3}{4}$	B.	<u>3</u> 5	C.	$\frac{3}{4}$	D.	<u>4</u> 5				
E. $\frac{3}{4}$	F.	$\frac{3}{10}$	G.	<u>2</u> 5	H.	$\frac{1}{2}$				
I. $\frac{1}{4}$	J.	<u>3</u> 5	K.	$\frac{1}{3}$	L.	$\frac{1}{4}$				
Section VI: Solving Addit	Section VI: Solving Addition & subtraction problems involving fractions with like denominators									
A. $\frac{3}{4}$ of a pizza	B.	$2\frac{3}{4}$ yards	C.	$l\frac{1}{3}$ cups	D.	$5\frac{1}{4}$ "				